10/11/2015 الثلاثاء د. عرفة محاضرة 6

* Polar Plot :-

Stability

① absolute Stability

② Relative Stability
- Bode Diagram
- polar plot
- Nyquist

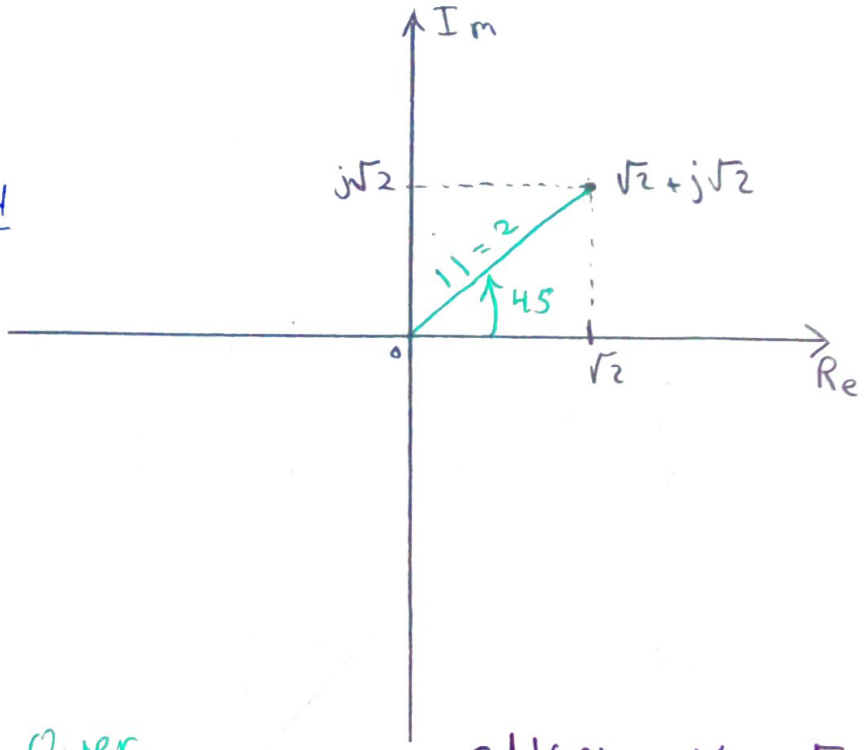
لتعقد وا كل ال Freq. Analysis

PM & GM

* one of the common method to study Relative Stability using Freq. Analysis technique.

* polar plot is a graph for o.L.T.F in polar plane which consists of two orthognal axis (Real & Img) like S - domain

* Polar plot represent the $|GH|$ and $\angle GH$ in polar plane for different values of ω.



⟹ Turn Over

$$GH(s) = \sqrt{2} + j\sqrt{2}$$

$$= \sqrt{2+2} \; \angle \tan^{-1}(\sqrt{2}/\sqrt{2})$$

$$= \underset{|1|}{2} \; \underset{\phi}{\angle 45^\circ}$$

① Map from S-domain to ω-domain

$$s \longrightarrow j\omega$$

$$GH(j\omega) = GH(s)\Big|_{s \to j\omega}$$

② obtain $|GH(j\omega)|$ and $\phi(\omega) = \angle GH(j\omega)$

③ Draw the $|GH(j\omega)|$ and $\phi(\omega)$ for different values of ω in polar plane.

Ex: $GH(s) = \dfrac{1}{(1+0.5s)}$

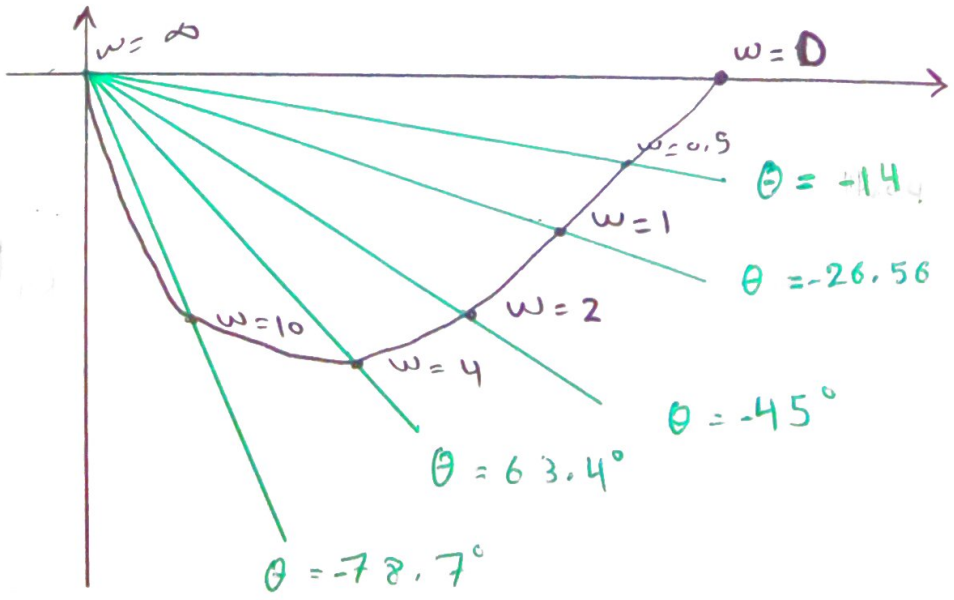
Draw the polar plot and find GM and PM

$$GH(j\omega) = \frac{1}{(1+0.5\,j\omega)}$$

$$|GH(j\omega)| = \frac{1}{\sqrt{1+(0.5\omega)^2}}$$

$$\phi(\omega) = \angle GH(j\omega) = 0 - \tan^{-1}(0.5\omega)$$

ω	0	0.5	1	2	10	∞	4
$\|GH(j\omega)\|$	1	0.97	0.89	$\frac{1}{\sqrt{2}} = 0.707$	$\frac{1}{\sqrt{26}} = 0.2$	0	$1/\sqrt{2} = 0.447$
$\phi(\omega)$	0	$-14.04°$	$-26.56°$	$-45°$	$-78.7°$	-90	$\phi = -63.4°$



يتبع سؤال ٤ Page 5

$$PM = 180 + \phi(\omega_{gc})$$

$$\Downarrow$$

$$|L| = 1$$

$$|L|_{dB} = 0$$

$PM = -ve \Rightarrow$ unstable

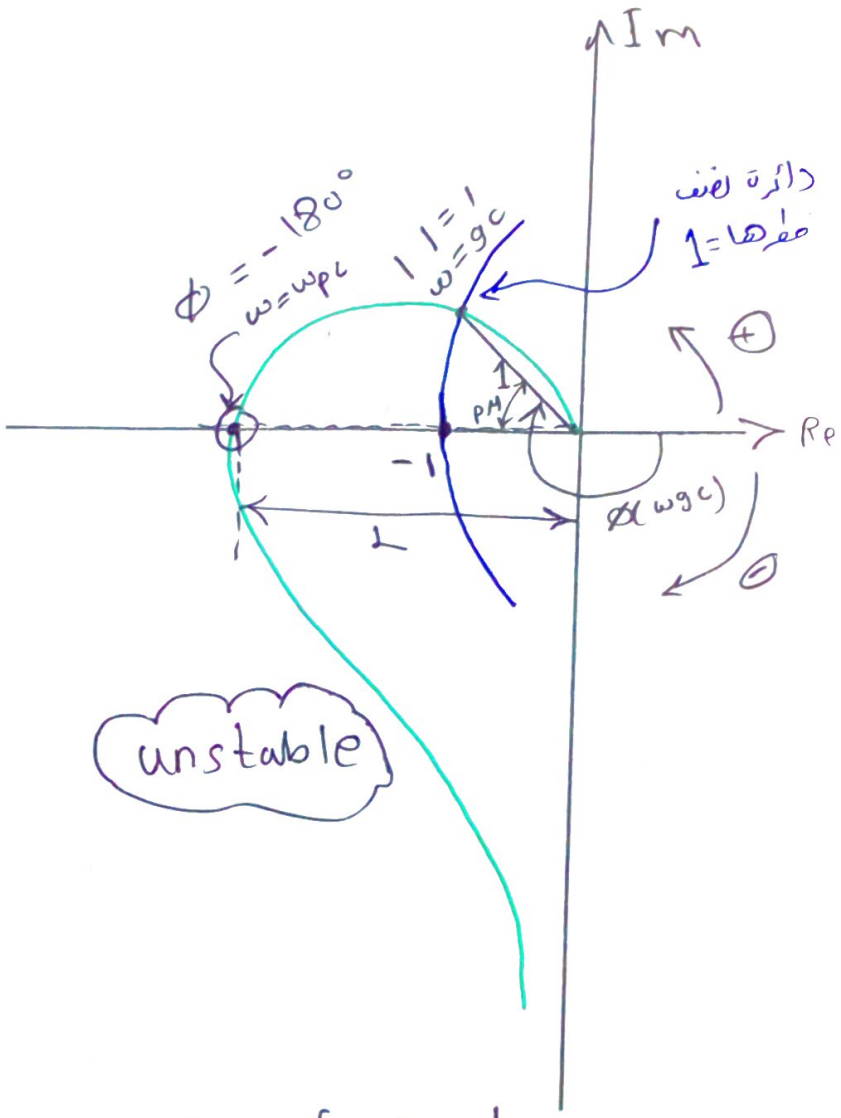
$$GM = \frac{1}{L}$$

$$GM_{dB} = 20 \log \frac{1}{L}$$

$$= -20 \log L$$

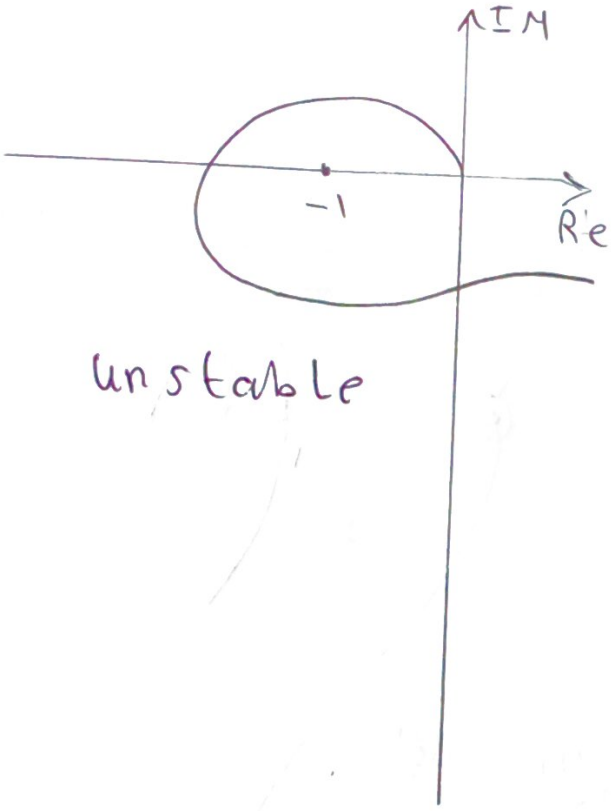
- GM
  - > 1 stable
  - $= 1$ Critically
  - < 1 unstable

- GM_{dB}
  - > 0 stable
  - $= 0$ Critically
  - < 0 unstable

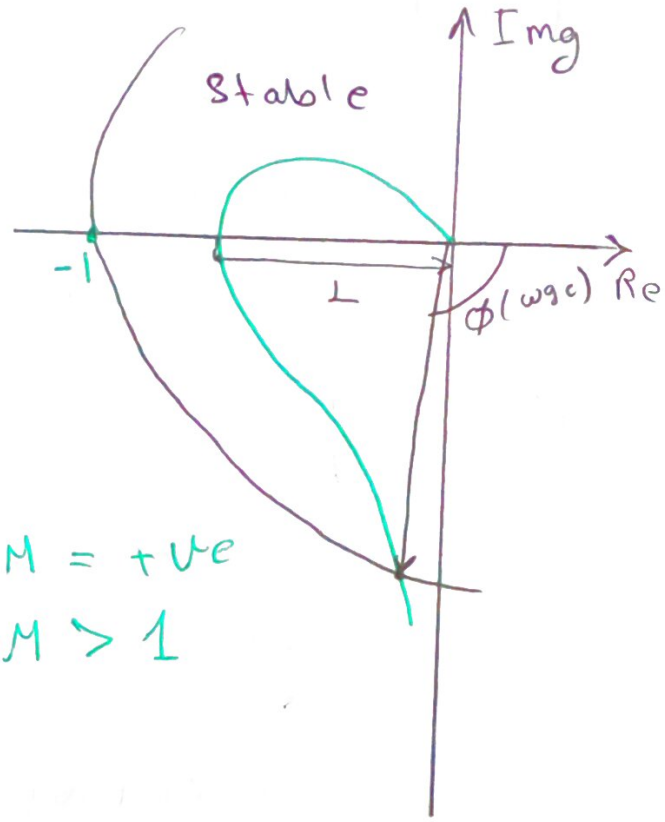


- لاحظ أنه الطول لوكان < 1 فدائما معلوبة عدد كبير وبالتالي يكون الـ System دائما unstable
- يعني لو تم احتواء (-1) داخل الرسمة يكون الـ System دائما unstable تماما هو في الرسم.

$\Rightarrow$ turn over

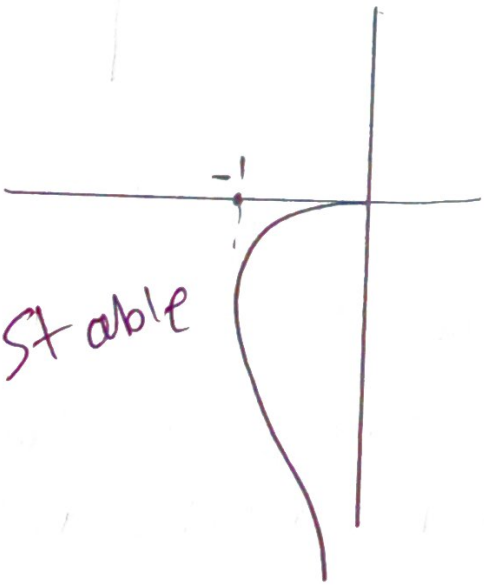


unstable

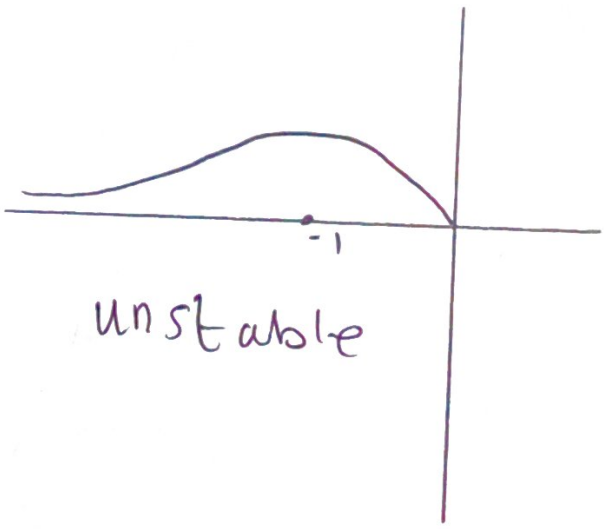


PM = +ve

GM > 1



Stable



unstable

* بالرجوع للمثال السابق (page 2) يتضح أن الـ System

⇓ stable

$PM = 180 + \phi(\omega_{gc})$

$\omega_{gc} \Rightarrow \omega$ at $|GH(j\omega)| = 1$

$\omega_{gc} = 0$

$PM = 180 + \phi(\omega_{gc} = 0)$

$= 180^\circ$ (+ve) stable

$\omega_{pc} \Rightarrow$ not defined

Ex 2: $GH(S) = \dfrac{1}{(2S+1)(0.5S+1)(S+1)}$

$S \longrightarrow j\omega$

$GH(j\omega) = \dfrac{1}{(2j\omega+1)(0.5j\omega+1)(j\omega+1)}$

$|GH(j\omega)| = \dfrac{1}{\sqrt{4\omega^2+1}\cdot\sqrt{0.25\omega^2+1}\cdot\sqrt{\omega^2+1}}$

$\phi(\omega) = \angle GH(j\omega) = -\tan^{-1}(2\omega) - \tan^{-1}(0.5\omega)$

$- \tan^{-1}(\omega)$

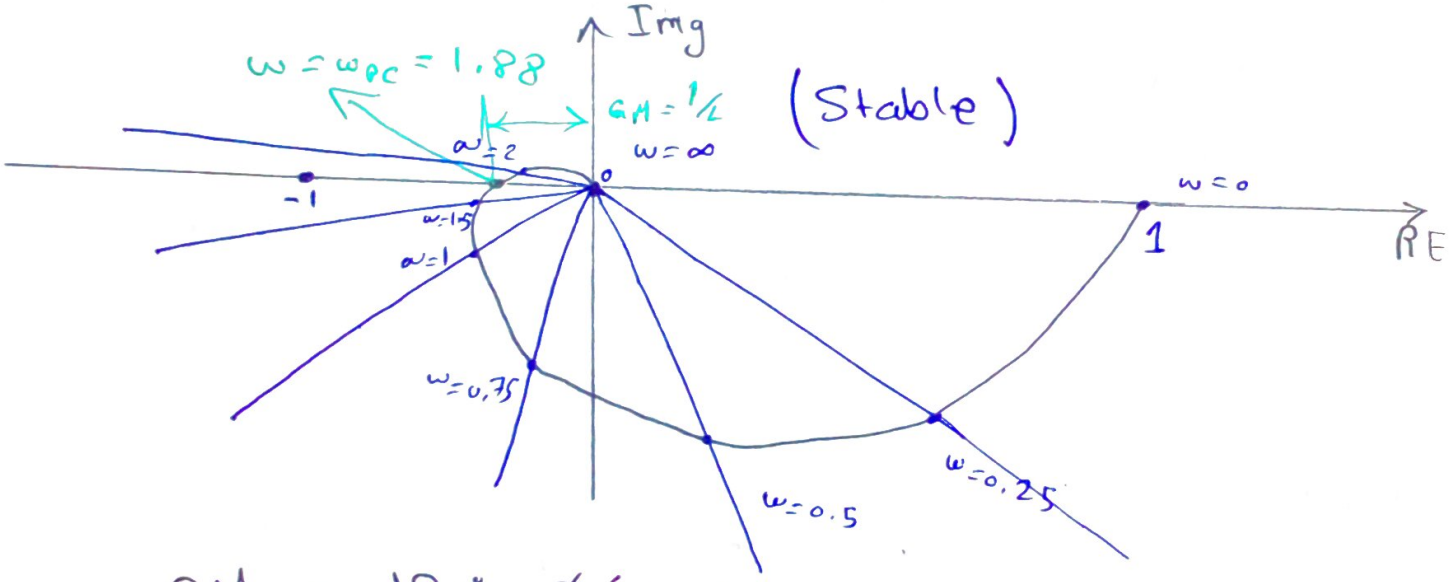
$\Rightarrow$ turn over

ω_{gc}

ω	0	0.25	0.5	0.75	1	1.5	2	5	∞
$\lvert GH \rvert$	1	0.86	0.614	0.415	0.28	0.14	0.077	0.0072	0
$\phi(\omega)$	0	-47.7°	-85.6°	-113.7°	-135	-164.7	-184.4	-231.2°	-270

$\omega_{pc} \simeq$

دائما الرسمة تنتهي عند نقطة الصفر (Zero)



$$PM = 180^\circ + \phi(\omega_{gc}) \rightarrow |1| = 1$$
$$= 180^\circ + \phi(\omega = 0)$$
$$= 180^\circ \checkmark \ (+ve) \ \text{stable}$$

$GM \Rightarrow \omega_{pc}$

ω at $\phi(\omega) = -180$

حاسبة أمبيا رياضيا أو رسوميا

Trial and error gives:

$\omega = 1.9 \Rightarrow \phi(\omega) = -184.4^\circ$

$\omega = 1.8 \Rightarrow \phi(\omega) = -177.4^\circ$

$\omega = 1.88 \Rightarrow \phi(\omega) = 180.325$

$\omega_{pc} \simeq 1.88$

$$GM = \frac{1}{L} = \frac{1}{|GH(j\omega)|}$$

$\omega = \omega_{pc} = 1.22$

$GM = 11.37 > 1 \Rightarrow$ Stable

$GM_{dB} = 20 \log GM = 20 \log(11.37) = 21.16$ dB

$GM > 0$ +ve $\Rightarrow$ stable

to get ω_{pc} Mathematically (not graphically)

$$\phi(\omega) = -\tan^{-1}(2\omega) - \tan^{-1}(0.5) - \tan^{-1}(\omega)$$

$\omega_{pc} = \omega$ at $\phi(\omega) = -180°$

$$+180 = +\tan^{-1}(2\omega_{pc}) + \tan^{-1}(\omega_{pc}) + \tan^{-1}(0.5\omega_{pc})$$

$$180 - \tan^{-1}(2\omega_{pc}) = \tan^{-1}(0.5\omega) + \tan^{-1}(\omega$$

$$\frac{\tan 180^\circ - 2\omega_{pc}}{1 + \tan 180^\circ * 2\omega_{pc}} = \frac{0.5\omega_{pc} + \omega_{pc}}{1 - \omega_{pc} * 0.5\omega_{pc}}$$

$$-2\omega_{pc} = \frac{1.5\omega_{pc}}{1 - 0.5\omega_{pc}^2}$$

$$1.5 = -2 + \omega_{pc}^2$$

$$\Rightarrow \omega_{pc}^2 = 3.5 \Rightarrow \omega_{pc} = \sqrt{3.5} = 1.870 \text{ rad/s}$$

$\Longrightarrow$ Turn Over

على الهامش

$$\tan(X \pm Y) = \frac{\tan X \pm \tan Y}{1 \mp \tan X \cdot \tan Y}$$

Ex 3: $GH(s) = \dfrac{1}{s(1+s)(0.5+s)}$ type 1

$s \to jw$

$$GH(jw) = \frac{1}{jw(1+jw)(0.5+jw)}$$

$$|GH(jw)| = \frac{1}{w\sqrt{1+w^2}\sqrt{0.25+w^2}}$$

$$\angle GH(jw) = -90^\circ - \tan^{-1}(w) - \tan^{-1}(2w)$$

w	0	0.25	0.5	0.75	1	1.5	2	∞
$\|GH\|$	∞	6.9	2.53	1.183	0.6	0.234	0.108	0
$\phi(w)$	-90	-102.53°	-161.56°	-183.18°	-198.43	-217	-229.4°	-270

$w_{pc} \Rightarrow w$ at $\phi = -180^\circ$

$w = 0.75 \Rightarrow \phi = 183.18^\circ$

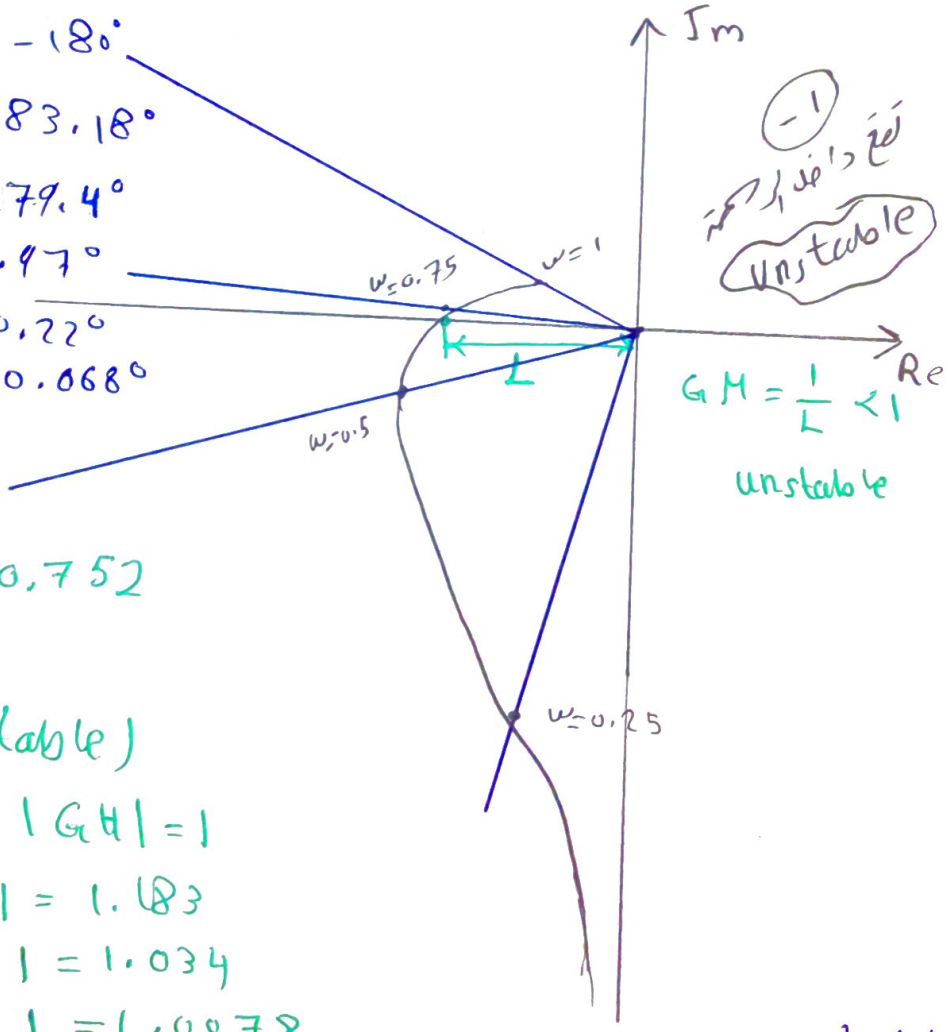
$w = 0.7 \Rightarrow \phi = 179.4^\circ$

$w = 0.72 \Rightarrow \phi = 180.97^\circ$

$w = 0.71 \Rightarrow \phi = 180.22^\circ$

$w = 0.708 \Rightarrow \phi = 180.068^\circ$

$w_{pc} \simeq 0.708$



$$GM = \frac{1}{|GH|} = 0.752$$

$GM < 1$ (unstable)

$w_{gc} \Rightarrow w$ at $|GH| = 1$

$w = 0.75 \Rightarrow |\,| = 1.183$

$w = 0.8 \Rightarrow |\,| = 1.034$

$w = 0.81 \Rightarrow |\,| = 1.0078$

$w_{gc} = 0.81$ rad/sec

$PM = 180 + \phi(w_{gc} = 0.81) = -7.32^\circ$ -ve

unstable ⇑